Sentence

Let $G = (V, T, P, S)$ be a Context-free Grammar.

A string $w \in (V \cup T)^*$ which is derivable from the start symbol S such that $S \overset{*}{\Rightarrow} w$ is called a sentence or sentential form of G.

Example

$$
\begin{aligned}
E &\Rightarrow E + E \\
&\Rightarrow id + E \\
&\Rightarrow id + E * E \\
&\Rightarrow id + id * E \\
&\Rightarrow \underbrace{id + id * id.}_{\text{Sentence of the Grammar.}}
\end{aligned}
$$

Types of Sentence or Sentential Forms

1) Left sentential form
2) Right sentential form.

Left Sentential Form

If the derivation is of the form $S \overset{*}{\Rightarrow} \alpha$, where at each step in the derivation process only a left most variable is replaced; then α is called Left Sentential form of G.

Grammar

$E \to E+E$

$E \to E*E$

$E \to (E)$

$E \to id$

Leftmost Derivation

$E \Rightarrow E+E$

$E \Rightarrow id + E$

$\Rightarrow id + E*E$

$\Rightarrow id + id * E$

$\Rightarrow id + id * id.$

are all left sentential forms of the Grammar

Right Sentential Form

If there is a derivative of the form $S \overset{*}{\Rightarrow} \alpha$, where at each step in the derivation process only a right most non-terminal is replaced then $\underline{\alpha}$ is called right sentential form of G.

Grammar

$E \to E+E$

$E \to E*E$

$E \to (E)$

$E \to id$

Rightmost derivation

$E \Rightarrow E + \underline{E}$

$\Rightarrow E + E*E$

$\Rightarrow E + E * id$

$\Rightarrow E + id * id$

$\Rightarrow id + id * id$

Right sentential forms.

Language generated by a Grammar

Def Let $G = (V, T, P, S)$ be a grammar. The language $L(G)$ generated by the grammar G is

$$L(G) = \{ w \mid S \overset{*}{\Rightarrow} w \text{ and } w \in T^* \}$$

where w is the string of terminals (may be ε) obtained from the start symbol S by applying various productions

e.g., $A \rightarrow a \mid aA$.

$L = \{a, aa, aaa, aaaa, \ldots\}$.

Q) Consider the grammar

$S \rightarrow aCa$

$C \rightarrow aCa \mid b$.

What is the language generated?

Sol. (I) $S \Rightarrow aCa$ using $S \rightarrow aCa$

$\Rightarrow aba$ $C \rightarrow b$

(II) $S \rightarrow aCa$ using $S \rightarrow aCa$

$\rightarrow aaCaa$ $C \rightarrow aCa$

$\rightarrow aaaCaaa$ $C \rightarrow aCa$

$\rightarrow aaabaaa$ $C \rightarrow b$.

$\Rightarrow a^n c a^n$ by applying $c \rightarrow aCa$ $n-1$ times

$\Rightarrow a^n b a^n$ $c \rightarrow b$.

$\therefore L(G) = \{ a^n b a^n \mid n \geq 1 \}$

Grammars for other languages

Q) Obtain a grammar to generate the following language:

$$L = \{a^n b^n : n \geq 0\}$$

Sol. To generate any number of a's

$$S \rightarrow \varepsilon \mid aS$$

For every a, we need to generate a b (by suffixing aS with b)

$$\therefore S \rightarrow aSb$$

$\therefore S \rightarrow \varepsilon \mid aSb$ is the required grammar.

Q) Obtain G for given $L = \{a^n b^n$ for $n \geq 1\}$

Sol. $S \rightarrow ab \mid aSb$

Q) Obtain a grammar G to generate set of all palindromes over $\Sigma = \{a, b\}$ (5)

Sol Recursive definition of a palindrome is

1. ϵ is a palindrome $\Rightarrow S \rightarrow \epsilon$
2. a and b are palindromes

$$S \rightarrow a \mid b$$

3. If w is a palindrome, then the string awa and bwb are palindromes

So, if S is a palindrome, aSa and bSb are also palindromes

$$S \rightarrow aSa \mid bSb$$

Putting everything together,

$$S \rightarrow \epsilon$$
$$S \rightarrow a \mid b$$
$$S \rightarrow aSa \mid bSb$$

Q) Obtain a grammar to generate

$$L = \{0^m 1^m 2^n \mid m \geq 1 \text{ and } n \geq 0\}$$

Sol :→

$$L = \underbrace{0^m 1^m}_{A} \underbrace{2^n}_{B}$$

$$S \rightarrow AB$$

- A should produce m number of 0's and 1's with minimum string 01.
- B should produce any number of 2's including ϵ

$\therefore \quad A \rightarrow 01 \mid 0A1$

$\quad\;\; B \rightarrow \varepsilon \mid 2B$

$\therefore \quad S \rightarrow AB$

$$\left.\begin{array}{l} S \rightarrow AB \\ A \rightarrow 01 \mid 0A1 \\ B \rightarrow \varepsilon \mid 2B \end{array}\right\} \text{required grammar}$$

Q) What is the language generated by the grammar

$S \rightarrow 0A \mid \varepsilon$

$A \rightarrow 1S$

Sol:→ $S \rightarrow \varepsilon \rightarrow$ null string.

$S \rightarrow 0A \quad (S \rightarrow 0A)$

$\rightarrow 01S \quad (A \rightarrow 1S)$

$\rightarrow 010A \quad (S \rightarrow 0A)$

$\rightarrow 0101S \quad (A \rightarrow 1S)$

$\rightarrow 0101 \quad (S \rightarrow \varepsilon)$

$L = \{ w \mid w \in \{01\}^* \}$

or

$L = \{ (01)^n \mid n \geq 0 \}$